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Resummation of Nonperturbative Corrections to the Lepton Spectrum in Inclusive $B \rightarrow X \ell \bar{\nu}$ Decays

Thomas Mannel and Matthias Neubert
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

We apply the operator product expansion to resum the leading non-perturbative corrections to the endpoint region of the lepton spectrum in inclusive semileptonic $B \rightarrow X_q \ell \bar{\nu}$ decays, taking into account a finite quark mass m_q in the final state. We show that both for $b \rightarrow c$ and $b \rightarrow u$ transitions, it is consistent to describe these effects by a convolution of the parton model spectrum with a fundamental light-cone structure function. The moments of this function are proportional to forward matrix elements of higher-dimension operators. The prospects for an extraction of the structure function from a measurement of the lepton spectrum are discussed.

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1 Introduction

The heavy quark expansion, i.e. a systematic expansion in powers of Λ/m_Q (we use Λ as a generic notation for a typical hadronic scale of the strong interactions), has by now become a widely used tool in the theoretical description of systems containing a heavy quark Q interacting with light degrees of freedom [1, 2, 3, 4]. Its application to exclusive decays of heavy mesons and baryons has been explored already in great detail [5]. Recently, the idea has been put forward to generalize the heavy quark expansion to obtain a QCD-based description of inclusive decays of heavy hadrons [6, 7, 8, 9, 10, 11]. Similarly to the case of deep-inelastic scattering, an operator product expansion is applied to the product of two local currents. The mass of the decaying quark provides the large momentum scale. The total decay rates can then be written as an expansion in inverse powers of m_Q . The operators appearing at leading order have dimension three and correspond to the free-quark decay. The matrix elements of the dimension-four operators vanish by the equation of motion, and thus the leading nonperturbative corrections arise from dimension-five operators and are of order $(\Lambda/m_Q)^2$. For the case of B -meson decays, they can be parameterized by two low-energy parameters λ_1 and λ_2 , which are related to the kinetic energy K_b of the b -quark inside the B -meson, and to the mass splitting between B - and B^* -mesons. They are defined as [12]

$$K_b = \frac{\langle \vec{p}_b^2 \rangle}{2m_b} = -\frac{\lambda_1}{2m_b}, \quad m_{B^*}^2 - m_B^2 = 4\lambda_2. \quad (1)$$

The operator product expansion has also been used to calculate differential distributions, such as the charged-lepton energy spectrum in inclusive semileptonic decays. In this case, the relevant large momentum scale is $Q = m_b v - q$, where q denotes the momentum transfer to the lepton pair, and v is the velocity of the decaying hadron. After integrating over the neutrino momentum, the operator product expansion is a combined expansion in powers of Λ/m_b and $\Lambda/(m_b - 2E_\ell)$, where E_ℓ is the lepton energy in the rest frame of the decaying hadron. Over most of the available phase space these parameters are of similar magnitude. However, close to the endpoint, i.e. for $(m_b - 2E_\ell)$ of order Λ , the second expansion parameter is of order unity, and a partial resummation of the expansion becomes necessary. For the lepton spectrum in charmless semileptonic $B \rightarrow X_u \ell \bar{\nu}$ decays, as well as

for the photon spectrum in the penguin-induced $B \rightarrow X_s \gamma$ decays, this resummation has been constructed in Refs. [13, 14, 15], neglecting the mass of the quark in the final state. It has been shown that the leading nonperturbative effects can be related to a universal structure function, which describes the distribution of the light-cone residual momentum of the heavy quark inside the decaying hadron. The purpose of this paper is to generalize this approach to the case where the mass of the quark in the final state cannot be neglected. For simplicity, we will work to leading order in perturbation theory. Radiative corrections, which have been calculated at the parton level in Ref. [16, 17, 18], should however be included before confronting our results with experimental data. We shall consider $B \rightarrow X_q \ell \bar{\nu}$ transitions and treat m_q as a free parameter. As we shall see, the presence of several mass scales leads to technical and conceptual complications. Let us define the dimensionless ratio

$$\rho = \frac{m_q^2}{m_b^2}. \quad (2)$$

It will be natural to distinguish the three cases where ρ is of order 1, Λ/m_b , and $(\Lambda/m_b)^2$, respectively. The first case is of no phenomenological importance and will not be considered in detail. The second case is relevant for $b \rightarrow c$ transitions, where $\rho \simeq 0.1$. The third case applies, e.g., when one studies the effect of a small constituent mass of the u -quark in $b \rightarrow u$ transitions.

The necessity of a resummation of the naive expansion in powers of Λ/m_b close to the endpoint is apparent from the result obtained in Refs. [7, 8, 9] for the lepton spectrum in $B \rightarrow X_q \ell \bar{\nu}$ decays. Let us define

$$y = \frac{2E_\ell}{m_b}, \quad \Gamma_b = \frac{G_F^2 |V_{qb}|^2 m_b^5}{192\pi^3}, \quad (3)$$

and divide the differential decay rate into two pieces:

$$\frac{1}{2\Gamma_b} \frac{d\Gamma}{dy} = F(y, \rho) \Theta(1 - y - \rho) + S(y, \rho). \quad (4)$$

The first part is the result obtained in the free-quark decay model. The function $F(y, \rho)$ is given by

$$F(y, \rho) = y^2 \left\{ 3(1 - \rho)(1 - R^2) - 2y(1 - R^3) \right\}; \quad R = \frac{\rho}{1 - y}. \quad (5)$$

In Fig. 1, we show $F(y, \rho)$ evaluated for $\rho = 0.08$, which is an appropriate value for $B \rightarrow X_c \ell \bar{\nu}$ transitions. The function $S(y, \rho)$ in (4) contains the nonperturbative corrections to a free-quark decay. The expression for this functions obtained by naively constructing the operator product expansion to next-to-leading order in Λ/m_b is

$$\begin{aligned}
S(y, \rho) = y^2 & \left\{ \frac{\lambda_1}{[m_b(1-y)]^2} (3R^2 - 4R^3) \right. \\
& - \frac{\lambda_1}{m_b^2(1-y)} (R^2 - 2R^3) - \frac{3\lambda_2}{m_b^2(1-y)} (2R + 3R^2 - 5R^3) \\
& + \frac{\lambda_1}{3m_b^2} [5y - 2(3-\rho)R^2 + 4R^3] \\
& + \frac{\lambda_2}{m_b^2} [(6+5y) - 12R - (9-5\rho)R^2 + 10R^3] \Big\} \Theta(1-y-\rho) \\
& + \mathcal{O}[(\Lambda/[m_b(1-y)])^3].
\end{aligned} \tag{6}$$

It is apparent that the operator product expansion gives an expansion in two parameters, Λ/m_b and $\Lambda/(m_b - 2E_\ell) = \Lambda/[m_b(1-y)]$. Over most of the kinematic region, the terms in $S(y, \rho)$ are of order $(\Lambda/m_b)^2$ or smaller. However, provided that the parameter ρ is of order Λ/m_b or smaller,¹ the expansion becomes singular in the endpoint region: when $(1-y)$ is of order Λ/m_b , terms of order $(\Lambda/[m_b(1-y)])^n$ in $S(y, \rho)$ become of order unity. In Fig. 2, we show $S(y, \rho)$ for $\rho = 0.08$, $\lambda_2 = 0.12 \text{ GeV}^2$, and the two cases $\lambda_1 = -0.1 \text{ GeV}^2$ and $\lambda_1 = -0.3 \text{ GeV}^2$. The low-energy parameter λ_2 can be extracted from the known value of the B^*-B mass splitting. The average kinetic energy, and with it the value of λ_1 , are not well-known, however. Considering the various theoretical arguments about this quantity that have been discussed in the literature [19, 20, 21], we consider the two choices given above as reasonable “small” and “large” values for λ_1 . Recently, Bigi et al. have derived the lower bound $-\lambda_1 \geq \frac{3}{2} \lambda_2$ in a quantum mechanical framework [15]. However, since λ_1 and λ_2 have a different dependence on the renormalization point, this result cannot be rigorous once short-distance corrections are taken into account. In fact, it is possible to construct an explicit counter-example to this bound using QCD sum rules [22].

¹When ρ is of order unity, $(1-y)$ cannot become small in the physical region, and there is no problem with (6).

The effect of nonperturbative corrections is very small over most of the kinematic region. Close to the endpoint, however, a sharp spike of height $\sim -\lambda_1/(m_b\rho)^2$ develops. The singular behavior of the expansion becomes even more obvious when one goes to higher orders; at order $1/m_b^3$, for instance, one encounters a δ -function at $y = 1 - \rho$. Clearly, one cannot believe the shape of the function $S(y, \rho)$ in this region. To get a more reliable description of the spectrum, the series in $\Lambda/[m_b(1 - y)]$ has to be resummed. In Ref. [13], it has been shown that when the mass m_q of the final-state quark is neglected, such a resummation can be performed and leads to a smooth, but rapidly varying, function. This so-called shape function is a genuinely nonperturbative object, which can be defined in terms of forward matrix elements of certain operators in the heavy quark effective theory. A complete resummation of the operator product expansion is, of course, a too complicated task. What can be achieved is a summation of the leading terms in the limit $m_b \rightarrow \infty$ with $m_b(1 - y)$ kept fixed.

In this paper we shall elaborate on this proposal and extend it to the case m_q not being zero. Taking the limit $m_b \rightarrow \infty$ with $m_b(1 - y)$ fixed, we find from (6)

$$S(y, \rho) \rightarrow \frac{\lambda_1 y^2}{[m_b(1 - y)]^2} (3R^2 - 4R^3) \Theta(1 - y - \rho) + \mathcal{O}[(\Lambda/[m_b(1 - y)])^3]. \quad (7)$$

Our goal will be to generalize this expression to all orders in $\Lambda/[m_b(1 - y)]$. In Sect. 2, we discuss in detail the structure of the operator product expansion and explain the significance of three different kinematic regions, which we will call the free-quark decay region, the endpoint region, and the resonance region. Our focus here is on the endpoint region, for which we construct an appropriate approximation that resums the leading singularities. We show that it is natural to distinguish the cases where the parameter ρ in (2) is of order Λ/m_b or $(\Lambda/m_b)^2$. We emphasize that there are, in the context of the operator product expansion, incalculable corrections to our results from bound-state effects in the final state. In the two cases considered above, they are suppressed by a factor $(\Lambda/m_b)^{1/2}$ or Λ/m_b , respectively. In Sect. 3, we apply our formalism to inclusive semileptonic B -meson decays and calculate the leading terms in the charged-lepton energy spectrum. We show that, in both cases, the spectrum can be written as a convolution of the free-quark decay rate with a fundamental light-cone structure function.

In Sect. 4, we illustrate our results using a simple model, which incorporates the known properties of the structure function and provides a description of the decay spectrum in terms of a single parameter. Sect. 5 deals with a discussion of possibilities to extract information about the structure function from experimental data on the lepton spectra in inclusive semileptonic B -decays. In Sect. 6, we summarize our results and give some conclusions.

2 Resummation of the Leading Endpoint Singularities

By the optical theorem, any inclusive decay rate can be related to the imaginary part of a transition operator T , which is defined in terms of the time-ordered product of two local operators. For the cases at hand, this correlator is of the form

$$T(q^2, v \cdot q) = -i \int d^4x e^{i(m_b v - q) \cdot x} \langle B(v) | T \left\{ \bar{b}_v(x) \Gamma_1 q(x), \bar{q}(0) \Gamma_2 b_v(0) \right\} | B(v) \rangle, \quad (8)$$

where q is the momentum transferred to the leptons, and Γ_i are combinations of Dirac matrices. The b -quark field $b_v(x)$ is related to the conventional field that appears in the QCD Lagrangian by a phase redefinition:

$$b_v(x) = \exp(-im_b v \cdot x) b(x). \quad (9)$$

This is appropriate to make explicit a trivial but strong dependence of the free quark field on the large mass m_b . Written in terms of the rescaled fields, the hadronic matrix element in (8) is free of large momentum scales. Hence, the relevant momentum transfer is

$$Q = m_b v - q. \quad (10)$$

Up to the difference between m_b and m_B , the variable Q^2 corresponds to the invariant mass of the hadronic final state. As long as Q^2 is large enough, it is legitimate to expand the correlator in a tower of local operators of increasing dimension, which are multiplied by coefficient functions that contain inverse powers of Q . In general, this will be an expansion in three large parameters: m_b , $v \cdot Q$, and Q^2 . Over most of phase space, Q^2 and $v \cdot Q$ scale with the heavy quark mass, and the operator product expansion reduces to an expansion in

powers of Λ/m_b . There is, however, a kinematic region where $v \cdot Q$ is of order m_b , but $\Lambda^2 \ll Q^2 \ll m_b^2$. In this case, it is consistent to work to leading order in Λ/m_b , but necessary to keep higher-order terms in Q^2 . In fact, as we shall see, one has to resum these terms to all orders.

To leading order in Λ/m_b , the heavy quark field b_v in (8) can be replaced by the corresponding two-component spinor h_v of the heavy quark effective theory [2]. Similarly, the physical B -meson states are replaced by the corresponding states in the $m_b \rightarrow \infty$ limit. At tree-level, the leading contribution to the correlator is obtained by contracting the light-quark fields:

$$T(q^2, v \cdot q) = \int d^4x e^{iQ \cdot x} \langle B(v) | T \{ \bar{h}_v(x) \Gamma_1 S_q(x, 0) \Gamma_2 h_v(0) \} | B(v) \rangle. \quad (11)$$

Here $S_q(x, 0)$ is the propagator of the q -quark in the background field of the light constituents of the B -meson. The Fourier transform of $S_q(x, 0)$ has the form

$$\begin{aligned} S_q(Q) &= \frac{1}{\not{Q} + i\not{D} - m_q + i\epsilon} \\ &= (\not{Q} + i\not{D} + m_q) \frac{1}{(Q^2 + 2iQ \cdot D - m_q^2 - \not{D} \not{D} + i\epsilon)}. \end{aligned} \quad (12)$$

Note that a derivative acting on the rescaled heavy quark field corresponds to the residual momentum $k = p_b - m_b v$. Since the residual momentum results from the interactions of the heavy quark with light degrees of freedom, it is of order Λ .

There is a lot of information that can be deduced from the structure of the background-field propagator. We will be interested in the region where the components of Q are large (of order m_b), since otherwise the operator product expansion breaks down. To leading order in Λ/m_b , we can then neglect the covariant derivative in the numerator, as well as the term containing two derivatives in the denominator. This gives

$$S_q(Q) = \frac{\not{Q} + m_q}{Q^2 + 2iQ \cdot D - m_q^2 + i\epsilon} + \mathcal{O}(\Lambda/m_b). \quad (13)$$

The term in the denominator containing the covariant derivative is of order Λm_b . To see whether it is important, we have to distinguish different kinematic regions. When $Q^2 \sim m_b^2$, the derivative term is suppressed by a

factor Λ/m_b . To leading order, the propagator corresponds to the propagator of a free quark with momentum Q . In this region, we thus recover the free-quark decay model up to small nonperturbative corrections. However, in the endpoint region, where $Q^2 - m_q^2 \sim \Lambda m_b$, it is not possible to neglect the derivative term. This is why an expansion in powers of $iQ \cdot D/Q^2$, which was applied in Refs. [7, 8, 9] to derive (6), becomes singular. Instead, one has to use (13) for the propagator in the endpoint region. For even smaller values $Q^2 - m_q^2 \sim \Lambda m_q$, the derivative term becomes the dominant term in the denominator, and the operator product expansion breaks down. This is the resonance region, where the invariant mass Q^2 of the hadronic final state is close to the mass-shell of the lowest-lying resonances containing a q -quark. The fact that the resonance region is parametrically smaller than the endpoint region implies that corrections to our results, which arise from bound-state effects in the final state, are suppressed in the $m_b \rightarrow \infty$ limit. This is fortunate, since these corrections are, as a matter of principle, not calculable using an operator product expansion.

Let us elaborate on this last point. To see to what level one is sensitive to bound-state corrections in the final state, we replace the quark mass m_q by some effective mass $(m_q + \delta)$, with δ of order Λ . The sensitivity to such effects depends upon the size of m_q . For the hypothetical case $m_q \sim m_b$, the corrections induced by $\delta \neq 0$ are of order Λm_b and thus of the same magnitude as the term containing the covariant derivative. In this case, the nonperturbative corrections to the free-quark decay model are incalculable. In the opposite limit $m_q \sim \Lambda$, corresponding to $\rho \sim (\Lambda/m_b)^2$, we find that the effects corresponding to $\delta \neq 0$ are always subleading. They are suppressed relative to the bound-state corrections in the initial state by a factor Λ/m_b . This shows that, in the analysis of the inclusive decays $B \rightarrow X_u \ell \bar{\nu}$ and $B \rightarrow X_s \gamma$ [13, 14], the effect of a non-vanishing u - or s -quark mass, even as large as a constituent mass, can be neglected. A very interesting case in between the above is the situation where $m_q^2 \sim \Lambda m_b$, corresponding to $\rho \sim \Lambda/m_b$. We argue that this case applies for $B \rightarrow X_c \ell \bar{\nu}$ decays, where $\rho \simeq 0.1$. Then the term in the propagator containing the derivative is of the same magnitude as ρ , and final-state corrections induced by $\delta \neq 0$ are again subleading. However, they are only suppressed by a factor $\Lambda/m_q \sim (\Lambda/m_b)^{1/2}$. We thus expect that these corrections are more important than in the case of decays into light (charmless) final states, and that the approach to the $m_b \rightarrow \infty$ limit will be slower.

The above discussion shows that, for the relevant cases $\rho \sim \Lambda/m_b$ and $\rho \sim (\Lambda/m_b)^2$, the propagator (13) can be used to resum the leading terms in the endpoint region. Corrections to it are suppressed in the $m_b \rightarrow \infty$ limit. It is possible to simplify the result further. To this end, we write $Q = (v \cdot Q)(n + \delta n)$ and choose the vectors n and δn such that n is a null-vector on the forward light cone satisfying $n^2 = 0$ and $n \cdot v = 1$, and thus $v \cdot \delta n = 0$. In the region where the derivative term in (13) is important, i.e. for $v \cdot Q$ of order m_b but Q^2 of order Λm_b , we have $n \cdot \delta n \sim \Lambda/m_b$. As long as we restrict ourselves to the leading term in the large- m_b limit, we can thus neglect δn . This leads to the final expression for the effective background-field propagator:

$$S_q(Q) \simeq \int dk_+ \delta(k_+ - iD_+) \frac{\not{Q} + m_q}{Q^2 + 2k_+ v \cdot Q - m_q^2 + i\epsilon}, \quad (14)$$

where we define $n \cdot D \equiv D_+$. In light-cone gauge, $n \cdot A \equiv A_+ = 0$, the operator iD_+ reduces to $i\partial_+$ and corresponds to the light-cone residual momentum k_+ of the heavy quark inside the b -meson. Note that the appearance of the null-vector n is closely connected to our assumption that m_q^2 is of order Λm_b or Λ^2 . For $m_q^2 \sim m_b^2$, one would have $Q^2 \sim m_b^2$ even in the endpoint region, and n would be a time-like vector [15]. However, as we have argued above, we do not believe that this case is of phenomenological importance. From now on, we shall not consider it any more.

Let us use this form of the propagator to calculate the leading contribution to the imaginary part of the correlator $T(q^2, q \cdot v)$. We can simplify the Dirac structure of the hadronic matrix element by using the identity

$$\bar{h}_v \Gamma h_v = \frac{1}{2} \text{Tr}(\Gamma P_v) \bar{h}_v h_v - \frac{1}{2} \text{Tr}(\gamma_\mu \gamma_5 P_v \Gamma P_v) \bar{h}_v \gamma^\mu \gamma_5 h_v, \quad (15)$$

which is valid for an arbitrary matrix Γ . Here $P_v = \frac{1}{2}(1 + \not{v})$ is a projector onto the large components. Since the matrix element of the axial vector current between B -meson states vanishes by parity invariance, we obtain for the leading term

$$\begin{aligned} \frac{1}{\pi} \text{Im} T(q^2, v \cdot q) &= -\frac{1}{4} \text{Tr} \left\{ \Gamma_1 (\not{Q} + m_q) \Gamma_2 (1 + \not{v}) \right\} \\ &\times \int dk_+ f(k_+) \delta(Q^2 + 2k_+ v \cdot Q - m_q^2), \end{aligned} \quad (16)$$

where

$$f(k_+) = \langle B(v) | \bar{h}_v \delta(k_+ - iD_+) h_v | B(v) \rangle \quad (17)$$

is the leading-twist structure function that determines the probability to find a b -quark with light-cone residual momentum k_+ inside the B -meson [14]. We use a mass-independent normalization of states such that $\langle B(v) | \bar{h}_v h_v | B(v) \rangle = 1$. It then follows that the structure function is normalized to unity:

$$\int dk_+ f(k_+) = 1. \quad (18)$$

It will later be convenient to introduce the Fourier transform of the structure function, which is given by the forward matrix element of a bilocal operator:²

$$\begin{aligned} \tilde{f}(t) &= \int dk_+ e^{-ik_+ t} f(k_+) \\ &= \langle B(v) | \bar{h}_v(0) P \exp \left[-i \int_0^t du A_+(un) \right] h_v(tn) | B(v) \rangle. \end{aligned} \quad (19)$$

This function obeys the normalization condition $\tilde{f}(0) = 1$. Note that the path-ordered exponential is absent in light-cone gauge.

Let us recall at this point some important properties of the structure function [13, 14]. The moments of $f(k_+)$ are given by forward matrix elements of leading-twist, higher-dimension operators in the heavy quark effective theory. They form a set of low-energy parameters A_n defined by

$$A_n = \int dk_+ k_+^n f(k_+) = i^n \tilde{f}^{(n)}(0) = \langle B(v) | \bar{h}_v (iD_+)^n h_v | B(v) \rangle, \quad (20)$$

where $\tilde{f}^{(n)}(0)$ is a short-hand notation for the n -th derivative of $\tilde{f}(t)$ evaluated at $t = 0$. Using the equation of motion of the heavy quark effective theory, one obtains for the first three moments

$$\begin{aligned} A_1 &= 0, \quad A_2 = -\frac{\lambda_1}{3}, \\ A_3 &= -\frac{v_\nu}{6} \langle B(v) | \bar{h}_v [D_\mu, g_s G^{\mu\nu}] h_v | B(v) \rangle, \end{aligned} \quad (21)$$

²The definition of a heavy quark structure function as the Fourier transform of the bilocal operator in (19) was used, in a different context, in Ref. [23].

where $g_s G^{\mu\nu} = i[D^\mu, D^\nu]$ is the gluon field-strength tensor, and λ_1 has been introduced in (1). Assuming that $0.1 \text{ GeV}^2 < -\lambda_1 < 0.3 \text{ GeV}^2$, we find that $180 \text{ MeV} < \sqrt{A_2} < 315 \text{ MeV}$. This quantity is related to the characteristic width of the endpoint region [13, 14]. Using the equation of motion for the gluon field, the moment A_3 may be written in terms of matrix elements of four-quark operators, which can be evaluated in the factorization approximation [15, 24]. This leads to the rough estimate

$$A_3 \approx -\frac{2\pi}{27} \alpha_s f_B^2 m_B \approx -(270 \text{ MeV})^3, \quad (22)$$

where we have assumed $f_B \approx 200 \text{ MeV}$ and $\alpha_s \approx 0.4$. Summarizing these results, we know that the light-cone structure function is centered around $k_+ = 0$, has a width of order 200-300 MeV determined by the average kinetic energy of the b -quark inside the B -meson, and most likely (if factorization holds approximately) has an asymmetry towards negative values of k_+ .

The support of the structure function can be deduced by observing that the total light-cone momentum fraction

$$x = \frac{(p_b)_+}{(p_B)_+} = \frac{m_b + k_+}{m_B} \quad (23)$$

must be bounded between 0 and 1. It follows that $-m_b \leq k_+ \leq m_B - m_b$. Since the structure function is defined in the heavy quark effective theory, corresponding to the $m_b \rightarrow \infty$ limit, this implies that

$$-\infty < k_+ \leq \bar{\Lambda}, \quad (24)$$

where $\bar{\Lambda}$ denotes the asymptotic value of the mass difference between a heavy meson and the heavy quark that it contains, and can be identified with the effective mass of the light degrees of freedom interacting with the heavy quark [25]. We expect that the support of $f(k_+)$ for negative values of k_+ is slightly larger (because of the asymmetry) but of the same order of magnitude, i.e. $f(k_+)$ should be exponentially small for $k_+ \ll -\bar{\Lambda}$.

3 Calculation of the Lepton Spectrum

Let us now proceed to calculate the charged-lepton spectrum in semileptonic B -decays. The matrices Γ_i in (16) are of the form $\gamma^\mu(1 - \gamma_5)$, and we obtain

for the leading contribution to the correlator

$$\frac{1}{\pi} \text{Im } T^{\mu\nu}(q^2, v \cdot q) = - \int dk_+ f(k_+) \delta(Q^2 + 2k_+ v \cdot Q - m_q^2) \times [Q^\mu v^\nu + Q^\nu v^\mu - g^{\mu\nu} v \cdot Q - i\epsilon^{\mu\nu\alpha\beta} Q_\alpha v_\beta]. \quad (25)$$

The next step is to express Q in terms of the lepton momentum q , and to contract $T^{\mu\nu}$ with the leptonic tensor. A straightforward calculation leads to the triple-differential decay rate

$$\frac{d^3\Gamma(B \rightarrow X_q \ell \bar{\nu})}{dq^2 d(v \cdot q) dE_\ell} = \frac{G_F^2 |V_{qb}|^2}{2\pi^3} (v \cdot q - E_\ell) (2m_b E_\ell - q^2) \int dk_+ f(k_+) \times \delta[q^2 - 2(m_b + k_+) v \cdot q + m_b^2 + 2k_+ m_b - m_q^2], \quad (26)$$

where $E_\ell = v \cdot p_\ell$ denotes the charged-lepton energy in the parent rest frame. To the order we are working, i.e. to leading order in the large- m_b limit, we can rewrite this expression in such a way that all dependence on m_b and k_+ comes through the combination

$$m_b^*(k_+) = m_b + k_+, \quad (27)$$

which we shall identify with the effective mass of the b -quark inside the B -meson. We thus observe that the leading bound-state corrections amount to averaging the parton-model rate for the decay of a quark with mass $m_b^*(k_+)$ over the distribution function $f(k_+)$. The free-quark decay model is recovered in the limit $f(k_+) \rightarrow \delta(k_+)$.

Next we integrate over $v \cdot q$, and then over q^2 in the kinematic region

$$0 \leq q^2 \leq 2E_\ell m_b^* (1 - R_*) ; \quad R_* = \frac{m_q^2}{m_b^*(m_b^* - 2E_\ell)}, \quad (28)$$

which follows from the requirement that $0 \leq q^2 \leq 4E_\ell E_\nu$. We obtain

$$\frac{d\Gamma}{dE_\ell} = \frac{G_F^2 |V_{qb}|^2}{12\pi^3} E_\ell^2 \int dk_+ f(k_+) \Theta[m_b^{*2} - m_q^2 - 2m_b^* E_\ell] \times \left\{ 3(m_b^{*2} - m_q^2)(1 - R_*^2) - 4m_b^* E_\ell (1 - R_*^3) \right\}. \quad (29)$$

This result has several interesting properties. We first note that the heavy quark mass m_b does no longer appear explicitly. For this reason, and in

particular when the focus is on the endpoint region, it would be unnatural to introduce the rescaled lepton energy $y = 2E_\ell/m_b$. Hence, we will hereafter present our results as function of the lepton energy E_ℓ , which is the quantity that is actually measured in experiments. Note, in particular, that (to the order we are working) the maximum value of the lepton energy is correctly reproduced. From the fact that $k_+^{\max} = \bar{\Lambda} = m_B - m_b$ according to (24), it follows that

$$(m_b^*)_{\max} = m_B, \quad E_\ell^{\max} = \frac{m_B}{2} \left(1 - \frac{m_q^2}{m_B^2} \right). \quad (30)$$

This should be compared to the kinematic endpoint $E_\ell^{\max} = (m_b/2)(1 - \rho)$ predicted by the free-quark decay model. The difference between the heavy quark mass and the physical B -meson mass is correctly accounted for in our approach. Note, however, that we are not able to account for the fact that, instead of the quark mass m_q , there should appear in (30) the mass of the lightest meson containing the q -quark. As discussed in Sect. 2, this effect is subleading, i.e. it vanishes in the $m_b \rightarrow \infty$ limit, whereas the difference between m_B and m_b remains.

For lepton energies not too close to the endpoint, the difference between m_b and the effective mass m_b^* becomes irrelevant, and (29) reduces to the result of the free-quark decay model, as given by the first term in (4). Since the first moment A_1 in (21) vanishes, the nonperturbative corrections in this region are of order $(\Lambda/m_b)^2$. Note, in particular, that

$$\int dk_+ f(k_+) [m_b^*(k_+)]^n = m_b^n \left\{ 1 - \frac{n(n-1)}{6} \frac{\lambda_1}{m_b^2} + \dots \right\}, \quad (31)$$

meaning that, up to second-order corrections, it is in fact the mass of the b -quark that matters in the main region of phase space [6].

It is clear from this discussion that substantial nonperturbative corrections show up only in the endpoint region, where $(m_b - 2E_\ell)$ is of order Λ , and the difference between m_b and m_b^* becomes important. We can separate these effects from the free-quark decay distribution by subtracting a term $\delta(k_+)$ from the structure function. In this way, we obtain the lepton spectrum as a sum of two terms, as shown in (4). Introducing then the rescaled lepton energy $y = 2E_\ell/m_b$ and the mass ratio $\rho = m_q^2/m_b^2$, and keeping only the leading contributions in the large- m_b limit, we obtain for the shape function

defined in (4)

$$S(y, \rho) = y^2 \int_{m_b(y-1+\rho)}^{\bar{\Lambda}} dk_+ [f(k_+) - \delta(k_+)] (1 - 3R_*^2 + 2R_*^3) + \text{less singular terms}, \quad (32)$$

where now

$$R_* = \frac{m_b \rho}{m_b(1-y) + k_+}. \quad (33)$$

This is the correct generalization of (7). If we would formally expand the structure function (17) as

$$f(k_+) = \delta(k_+) + \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} A_n \delta^{(n)}(k_+) \quad (34)$$

and truncate the series at $n = 2$, we would recover the singular term shown in (7). Such a truncation is not justified, however, since every term in (34) is of the same magnitude.

For completeness, we note that the form of the convolution (29) simplifies in the limit $m_q \rightarrow 0$, which is relevant for $B \rightarrow X_u \ell \bar{\nu}$ decays. One obtains

$$\begin{aligned} \frac{d\Gamma}{dE_\ell} &= \frac{G_F^2 |V_{qb}|^2}{12\pi^3} E_\ell^2 \int dk_+ f(k_+) \Theta(m_b^* - 2E_\ell) m_b^* (3m_b^* - 4E_\ell) \\ &\simeq \frac{G_F^2 |V_{qb}|^2}{12\pi^3} m_b E_\ell^2 (3m_b - 4E_\ell) \int_{2E_\ell - m_b}^{\bar{\Lambda}} dk_+ f(k_+). \end{aligned} \quad (35)$$

This agrees with the result obtained in Refs. [13, 14, 15].

4 A Realistic Model

At this point, it is instructive to illustrate our results with a simple, but realistic model. To this end, we propose the following one-parameter ansatz for the light-cone structure function:

$$f(k_+) = \frac{32}{\pi^2 \bar{\Lambda}} (1-x)^2 \exp \left\{ -\frac{4}{\pi} (1-x)^2 \right\} \Theta(1-x); \quad x = \frac{k_+}{\bar{\Lambda}}, \quad (36)$$

where $\bar{\Lambda} = m_B - m_b$ is treated as a free parameter. Below we shall use the value $\bar{\Lambda} = 0.57$ GeV, which is predicted by QCD sum rules [26, 27]. Our model structure function is shown in Fig. 3. It obeys all requirements that have been pointed out in Sect. 2. The support of $f(k_+)$ is limited to values $k_+ < \bar{\Lambda}$. The integral over the structure function is normalized to unity, and the first moment vanishes. The higher moments are proportional to powers of $\bar{\Lambda}$. In particular, we obtain

$$\begin{aligned} A_2 &= -\frac{\lambda_1}{3} = \left(\frac{3\pi}{8} - 1\right) \bar{\Lambda}^2 \simeq (0.42 \bar{\Lambda})^2, \\ A_3 &= -\left(2 - \frac{5\pi}{8}\right) \bar{\Lambda}^3 \simeq -(0.33 \bar{\Lambda})^3. \end{aligned} \quad (37)$$

For $\bar{\Lambda} = 0.57$ GeV, we find $\lambda_1 \simeq -0.17$ GeV² and $A_3 \simeq -(190 \text{ MeV})^3$. These numbers agree well with our estimates in Sect. 2. Given these low-energy parameters, we can compute the b -quark mass from the expansion [12]

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b^2} + \dots \quad (38)$$

In our model, we obtain $m_b \simeq 4.71$ GeV. Similarly, we find $m_c \simeq 1.35$ GeV for the charm-quark mass, and hence $\rho \simeq 0.08$. We shall use this set of parameters throughout the paper.

In Fig. 4a, we show the lepton spectrum $(1/\Gamma_b) d\Gamma/dE_\ell$ obtained from (29) in comparison with the free-quark decay distribution. The difference between the two spectra is shown in Fig. 4b. It corresponds to the shape function, which describes the leading nonperturbative corrections to the free-quark decay model. For the ease of comparison with Fig. 2, we have multiplied the vertical scale by $m_b/4$ in order to comply with the definition of $S(y, \rho)$ in (4). In order to convert the horizontal scale to the variable y , one would have to multiply E_ℓ by $2/m_b$. We find that the shape function is indeed sizable only in the endpoint region. Comparing the result of our resummation to the singular form of $S(y, \rho)$ given in (6) and shown in Fig. 2, we see that the resummation has eliminated the unrealistic spike at the endpoint. What remains is a smooth function, which is rapidly varying on scales of order $\Delta y \sim 1$. In other words, the decay probabilities have been redistributed such that the height of $S(y, \rho)$ is strongly reduced; however, and most importantly, the shape function now extends beyond the parton-model endpoint (which

would correspond to 2.17 GeV). Needless to say, the precise shape of $S(y, \rho)$ depends on the form of the structure function $f(k_+)$. In the following section, we shall discuss some strategies how to extract the structure function from data.

In Fig. 5, we show the corresponding spectra for charmless $B \rightarrow X_u \ell \bar{\nu}$ decays, setting $\rho = 0$. The nonperturbative effects in the endpoint region become more pronounced, due to the fact that the free-quark decay distribution ends with a step-function in this case.

5 Extraction of the Structure Function

In this section, we briefly discuss how one could, in principle, extract information about the structure function $f(k_+)$ from experimental data on inclusive semileptonic B -decays. We should mention from the beginning that our discussion will be incomplete in that it neglects radiative corrections as well as the effects of experimental uncertainties. Both could be important. It is also clear from the results of Refs. [14, 15] that a much better place to extract the structure function would be the photon spectrum in inclusive $B \rightarrow X_s \gamma$ decays. However, in view of the fact that this spectrum will be very hard to measure in the near future, whereas very detailed data for semileptonic decay spectra already exist [28], we think it is worthwhile to consider possibilities to extract at least some relevant information from semileptonic decays as well.

We start with a discussion of the charmless decays $B \rightarrow X_u \ell \bar{\nu}$. They are particularly simple, since the nonperturbative corrections in (35) are contained in an integral over the structure function. Up to an overall normalization factor (which depends on V_{cb} and m_b), one can directly extract the function $F(E_\ell)$ defined as

$$F(E_\ell) = \int_{2E_\ell - m_b}^{\bar{\Lambda}} dk_+ f(k_+) \propto \frac{1}{E_\ell^2 (3m_b - 4E_\ell)} \frac{d\Gamma}{dE_\ell} \quad (39)$$

from a measurement of the lepton spectrum. The normalization can be fixed by observing that $F(E_\ell)$ must approach unity when $m_b - 2E_\ell \gg \Lambda$. Using the definition (17) of the structure function, we obtain

$$F(E_\ell) = \langle B(v) | \bar{h}_v \Theta(m_b - 2E_\ell + iD_+) h_v | B(v) \rangle. \quad (40)$$

The derivative of $F(E_\ell)$ with respect to the lepton energy gives the structure function evaluated at $k_+ = 2E_\ell - m_b$:

$$F'(E_\ell) = -2 f(2E_\ell - m_b). \quad (41)$$

Given a measurement of $F(E_\ell)$, one can extract the moments A_n of the structure function, which have been defined in (20), by integration with appropriate weight functions. One finds that [13]

$$\int dE_\ell (2E_\ell - m_b)^n [F(E_\ell) - \Theta(2m_b - E_\ell)] = \frac{A_{n+1}}{2(n+1)}. \quad (42)$$

The fact that $A_1 = 0$ can be used to fix the value of m_b in the step-function. In practice, the presence of several sources of experimental and theoretical uncertainties (in particular, radiative corrections and corrections of order Λ/m_b , which we neglect in the above expressions) will probably limit this extraction method to the first few moments.

Let us now turn to the case of $B \rightarrow X_c \ell \bar{\nu}$ decays, where the form of the convolution integral (29) is more complicated. Let us assume that the parameters m_b and m_c have been extracted from a fit of the spectrum to the free-quark decay model in the region far away from the endpoint. One can then write the observed lepton spectrum in terms of the parameter

$$\varepsilon = 2E_\ell - m_b. \quad (43)$$

To leading order in the large- m_b limit, the differential decay rate (29) can be rewritten in the form of a convolution of the structure function $f(k_+)$ with the parton-model distribution function $G(\varepsilon)$,

$$\frac{1}{E_\ell^2} \frac{d\Gamma}{dE_\ell} \equiv F(\varepsilon) = \int dk_+ f(k_+) G(\varepsilon - k_+), \quad (44)$$

where, according to (5),

$$G(\varepsilon) \propto \Theta(-m_b\rho - \varepsilon) \left\{ 3m_b(1 - \rho)(1 - R^2) - 2(m_b + \varepsilon)(1 - R^3) \right\} \quad (45)$$

up to an overall factor, and $R = -m_b\rho/\varepsilon$. Let us now introduce the Fourier transforms

$$\begin{aligned} \tilde{F}(t) &= \int d\varepsilon e^{-i\varepsilon t} F(\varepsilon), \\ \tilde{G}(t) &= \int d\varepsilon e^{-i\varepsilon t} G(\varepsilon). \end{aligned} \quad (46)$$

Using (19), we find that

$$\tilde{f}(t) = \frac{\tilde{F}(t)}{\tilde{G}(t)}, \quad (47)$$

i.e., by taking the ratio of the Fourier transforms of the observed lepton spectrum and of the parton-model distribution, one can in principle extract the matrix element of the bilocal operator in (19). The Fourier transform of $\tilde{f}(t)$ gives the structure function. Note that the normalization of the spectra is irrelevant in this context, since we know the normalization of $\tilde{f}(t)$ at the origin: $\tilde{f}(0) = 1$.

Given an experimental determination of $\tilde{F}(t)$, one can in principle compute the low-energy parameters A_n using (20), i.e.

$$A_n = \left(i \frac{\partial}{\partial t} \right)^n \left(\frac{\tilde{F}(t)}{\tilde{G}(t)} \right) \Big|_{t=0}. \quad (48)$$

In practice, such an analysis will again most likely be limited to the first few moments.

A less ambitious approach that could be taken is to rely on a theory-inspired ansatz for the structure function $f(k_+)$, which should depend upon few parameters with a well-defined physical meaning. The goal would be to extract these parameters from a fit to experimental data. This procedure is familiar from the present analysis of inclusive decay spectra in the context of the phenomenological model of Altarelli et al. [29]. We believe that a reasonable parameterization of $f(k_+)$ should contain (i) the parameter $\bar{\Lambda}$, which determines the gap between the parton-model endpoint and the physical endpoint of the lepton spectrum, (ii) the parameter λ_1 , which is proportional to the width of the endpoint region, and (iii) an asymmetry parameter, which is related to the third moment A_3 of the distribution function. An extraction of these fundamental quantities, even if it is affected by substantial uncertainties, would be most desirable.

6 Conclusions

In the endpoint region of the lepton spectrum in inclusive semileptonic decays of B -mesons, a naive operator product expansion in powers of Λ/m_b breaks down. The reason is that in this region of phase space, there emerges a second

expansion parameter, $\Lambda/(2m_b - E_\ell)$, which is much larger than Λ/m_b . In this case, a partial resummation of the operator product expansion is necessary before the theoretical results can be compared with data. This resummation is such that one sums all contributions of order $[\Lambda/(2m_b - E_\ell)]^n$, keeping however only the leading terms in Λ/m_b . In many respects, it resembles the summation of leading-twist contributions in deep-inelastic scattering.

For the cases of $B \rightarrow X_u \ell \bar{\nu}$ and $B \rightarrow X_s \gamma$ decays, where the mass of the quark in the final state can be neglected, it has been shown in Refs. [13, 14, 15] that the leading nonperturbative contributions close to the endpoint may be resummed into a light-cone structure function $f(k_+)$, which gives the probability to find a b -quark with light-cone residual momentum k_+ inside the B -meson. This function is defined in terms of forward matrix elements in the heavy quark effective theory. It is thus independent of m_b and has a universal character. In the present paper, we have shown that the same structure function describes the inclusive decays in the presence of a finite quark-mass in the final state, provided that the ratio $\rho = (m_q/m_b)^2$ is consistently treated as being of order Λ/m_b . In this case, however, the nonperturbative corrections to the leading behavior are of order $(\Lambda/m_b)^{1/2} \sim \Lambda/m_q$. Some of these corrections are due to hadronization effects in the final state and appear to be incalculable in the context of the operator product expansion. Numerically, they could be quite significant. Yet, since $\rho \simeq 0.08$ for case of $B \rightarrow X_c \ell \bar{\nu}$ decays, we argue that our results should apply and should provide a correct description of the leading nonperturbative effects. We should point out that a different position was taken by Bigi et al. [15], where it was argued that $b \rightarrow c$ transitions are close to the so-called “small velocity limit”, where $m_b - m_c$ is assumed to be much smaller than m_b , corresponding to $\rho \sim 1$. In this case, even the leading effects are no longer described by the universal light-cone distribution function. Which of the two cases is closer to reality remains to be seen. We note, however, that the experimental fact that semileptonic B -decays are not saturated by the exclusive modes $B \rightarrow D^{(*)} \ell \bar{\nu}$ indicates that the “small velocity limit”, in which these two modes would saturate the total rate, can only be a rather crude approximation.

In both cases, $B \rightarrow X_u \ell \bar{\nu}$ and $B \rightarrow X_c \ell \bar{\nu}$ decays, the lepton spectrum can be written as a convolution of the free-quark decay distribution with the universal structure function $f(k_+)$. We have shown that the effect of the momentum distribution of the heavy quark inside the meson can be understood in terms of an effective mass $m_b^* = m_b + k_+$, which deter-

mines the decay kinematics. Over most of phase space, the free-quark decay distribution is slowly varying on scales of order Λ , whereas the light-cone distribution function is sharply peaked around $k_+ \sim 0$ with an intrinsic width of order Λ . In this case, the convolution reproduces the parton model up to small nonperturbative corrections of order $(\Lambda/m_b)^2$. In the endpoint region, however, the free-quark decay distribution falls steeply, and the convolution becomes sensitive to the details of the structure function. This leads to large, genuinely nonperturbative effects in the lepton spectrum, which may all be attributed to bound-state corrections in the initial state. In particular, as one approaches the endpoint, the effective mass approaches the mass of the physical B -meson, and one recovers the correct position of the maximum lepton energy.

We have illustrated the effects of nonperturbative corrections using a simple one-parameter model, which however includes many of the ingredients of a more sophisticated description. For simplicity, and since our main focus in this paper was to investigate the effects of bound-state corrections, we have not included in our discussion perturbative QCD corrections from the emission of real and virtual gluons. Such effects have been considered in Refs. [16, 17, 18], and more recently in Refs. [15, 30]. For inclusive decays into final states containing light quarks, their interplay with the nonperturbative corrections considered here is rather intricate because of the presence of large Sudakov double-logarithms. In particular, in $B \rightarrow X_u \ell \bar{\nu}$ decays the simple factorization of bound-state corrections into an integral over $f(k_+)$ [see (35)] is replaced by a more complicated convolution of the structure function with a hard-scattering amplitude. For the case of $B \rightarrow X_c \ell \bar{\nu}$ decays, however, such effects are known to be less severe. Nevertheless, further investigation of radiative corrections is necessary to put our formalism on a more quantitative basis.

In the last part of the paper, we have discussed some possible approaches that could be taken to extract information about the structure function from semileptonic decay spectra. We are aware that such an analysis will be complicated, due to various theoretical and experimental limitations. It is likely that the most promising approach will be to make a theory-motivated ansatz for the structure function that contains few physical parameters, and to extract these parameters from a fit to data. We are confident that it should be possible to obtain an (approximate) determination of the first two non-trivial moments of $f(k_+)$. They are proportional to the average kinetic

energy of the b -quark inside the B -meson, and to the asymmetry parameter A_3 . These fundamental parameters are of sufficient interest to try such an analysis using the existing data on $B \rightarrow X_c \ell \bar{\nu}$ decays.

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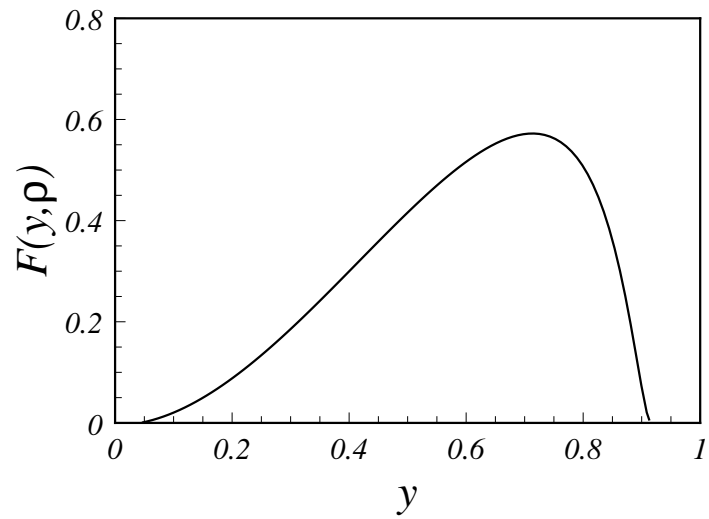


Figure 1: Shape of the lepton spectrum predicted in the free-quark decay model. We use $\rho = (m_c/m_b)^2 = 0.08$.

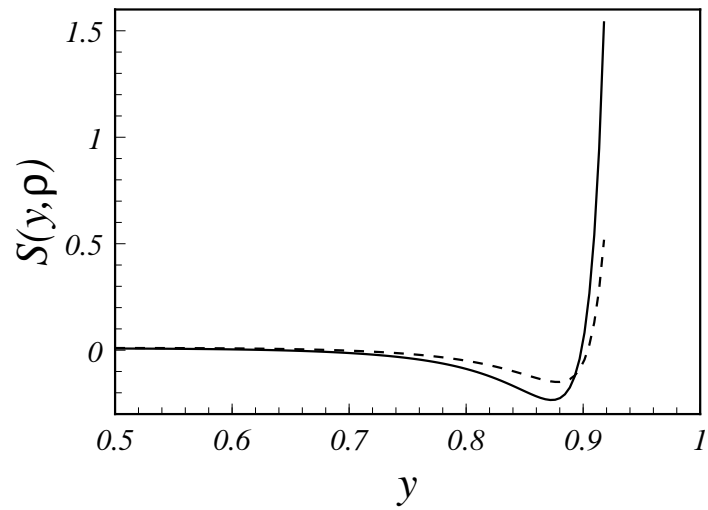


Figure 2: Nonperturbative corrections to the lepton spectrum obtained using a naive operator product expansion. The solid line corresponds to $\lambda_1 = -0.3 \text{ GeV}^2$, the dashed one to $\lambda_1 = -0.1 \text{ GeV}^2$.

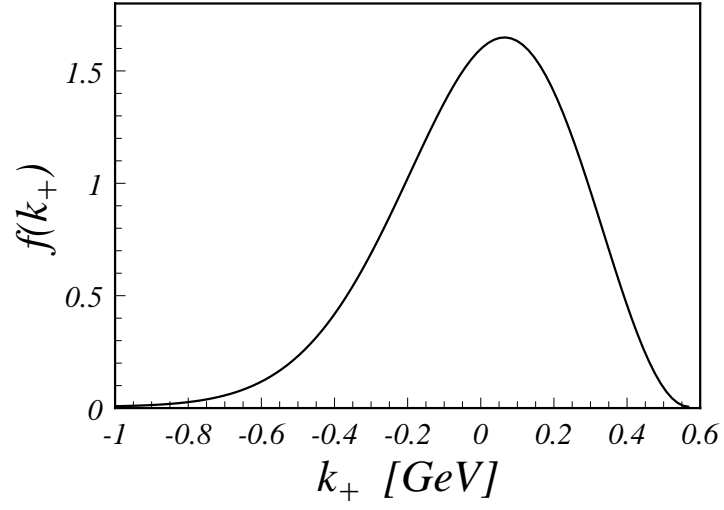


Figure 3: Model ansatz (36) for the structure function $f(k_+)$, evaluated for $\bar{\Lambda} = 0.57$ GeV.

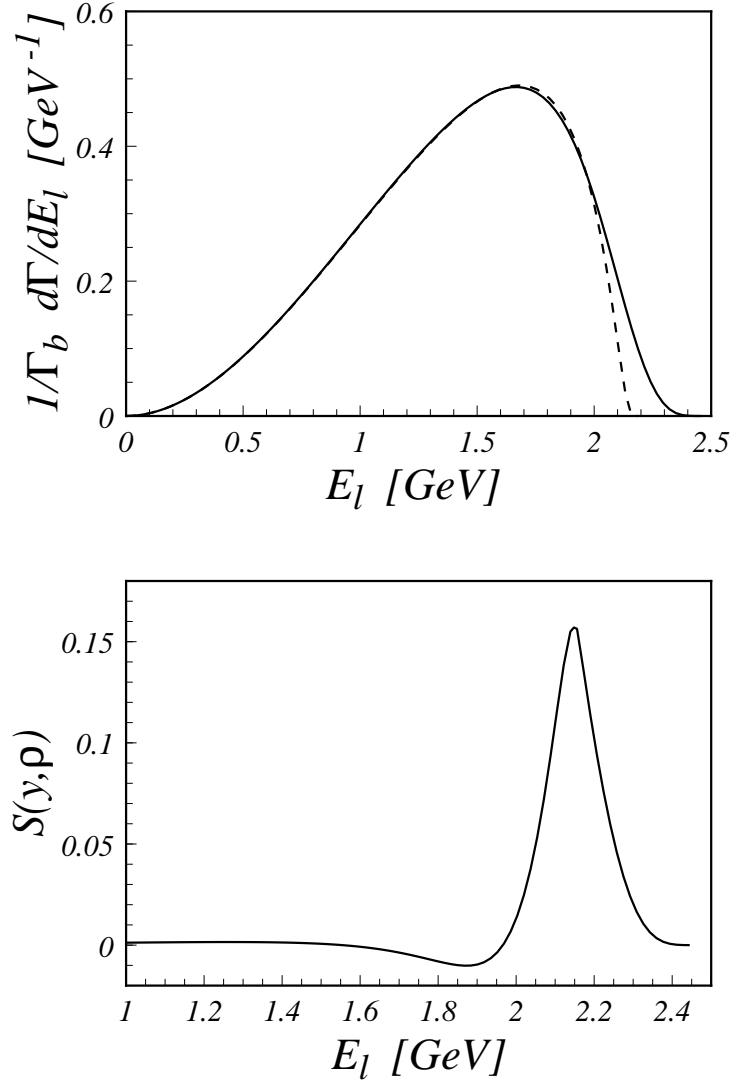


Figure 4: (a) Charged-lepton spectrum $(1/\Gamma_b) \, d\Gamma/dE_\ell$ in $B \rightarrow X_c \ell \bar{\nu}$ decays. The solid line is obtained from the convolution in (29) using the ansatz (36) for the structure function. The dashed line shows the prediction of the free-quark decay model. (b) The shape function $S(y, \rho)$, which is obtained from the difference of the two curves in (a).

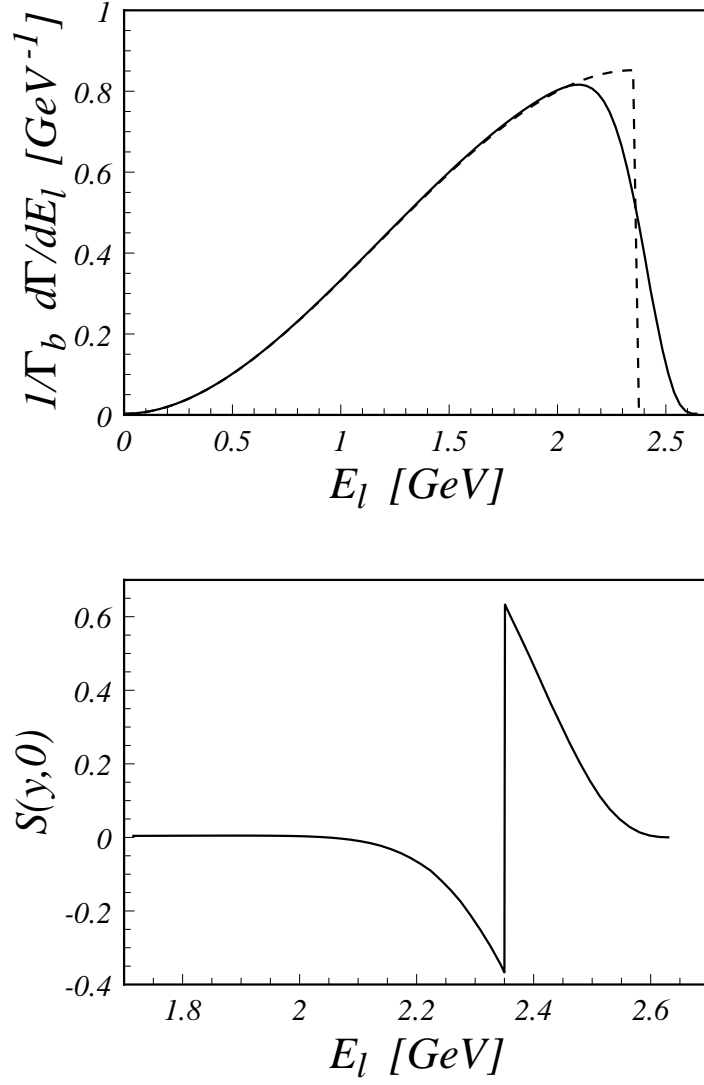


Figure 5: Same as Fig. 4, but for $B \rightarrow X_u \ell \bar{\nu}$ decays.

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